STABILITY OF ROUGHLY DISPERSE VERTICAL FLOWS

Sh. K. Kapbasov

UDC 532.546

Conditions have been found for neutral stability of vertical flows of suspended large particles. Characteristics of disturbances with a maximum growth increment in the instability region are considered.

Because of the nonlinear relation between the force of hydrodynamic interaction and local concentration, the fluctuation motion of disperse systems can retard the growth of the amplitude of kinematic waves arising in the medium. In the range of moderate concentrations the shock viscosity in a system of large particles does not give rise to a parametric region of stability, which stimulates naturally the formation of concentration discontinuities in the suspension and the appearance of gas cavities in the solid phase during fluidization [1-3]. Such properties of gas suspensions were studied in experiments and even under conditions where the effect of developing instability of the initially homogeneous flow was mainly one-dimensional [4]. Manwhile, some especially urgent problems that have great practical importance, including the scale effect, remain little studied. The scale effect is associated with the phenomenon of increasing instability in industrial installations with geometry similar to that of laboratory facilities. Attempts to investigate these problems have been made, but the reported agreement between theory and experiment was explained using empirical constants and relations. Meanwhile, the lack of a reliable rheological model greatly reduces substantially the possibility of using calculations which are often significant for a rather narrow range of parameters characterizing the flow [7].

In the present work, stability of vertical flows is considered in a wide range of these parameters within the theory suggested in [8, 9]. Rheological properties and characteristics of a disperse flow are developed under the action of random fluctuations of colliding particles such as these taking place in molecular gases. In this case the vertical flow of the system of particles is governed by the ordinary conservation equations [8]. Adding the mass equation for the liquid phase to them and neglecting relatively small additions to the complete phase flows caused by fluctuations as well as by fluctuation energy transfer and energy dissipation in collisions of the particles, we can write

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho w)}{\partial x} = 0 ; \quad \frac{\partial \rho}{\partial t} - \frac{\partial (\varepsilon v)}{\partial x} = 0 ; \quad \varepsilon = 1 - \rho , \tag{1}$$

$$\rho \left(\frac{\partial}{\partial t} + w \frac{\partial}{\partial x} \right) w = - \frac{\partial}{\partial x} \left(\frac{P_1}{d_1} \right) + \frac{\partial}{\partial x} \left(\frac{\mu_1}{d_1} \frac{\partial w}{\partial x} \right) +$$

$$+ \rho \left[\frac{K(\rho)}{\kappa a} u^2 - \varepsilon \left(1 - \frac{1}{\kappa} \right) g + \left(\frac{\varepsilon}{\kappa} + \rho \right) \left(\frac{\partial}{\partial t} + w \frac{\partial}{\partial x} \right) w \right] , \quad \kappa = \frac{d_1}{d_0} , \tag{2}$$

$$K(\rho) = \frac{3}{8} \xi \left(\frac{1 - \rho}{1 - 1.17 \rho^{2/3}} \right)^2 ,$$

$$\frac{3}{2} \rho \left(\frac{\partial}{\partial t} + w \frac{\partial}{\partial x} \right) \theta = - \frac{P_1}{d_1} \frac{\partial w}{\partial x} + 4 \rho \frac{u K(\rho)}{\kappa a} \sqrt{\theta} \left(\sqrt{\theta^*} - \sqrt{\theta} \right) . \tag{3}$$

Here v and w are the average velocities of the liquid and particles; ρ is the average volume concentration of the particles in the flow; d_0, d_1 are the densities of the liquid and particle material; θ is the effective temperature of

---

Ural State University, Ekaterinburg, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 68, No. 4, pp. 542-551, July-August, 1995. Original article submitted December 29, 1993.

 1062-0125/95/6804-0442$12.50

the pseudogas of particles; P_1 and μ_1 are the normal stress and the dynamic viscosity of the system of suspended particles.

By analogy with [10], the momentum equation for the continuous phase containing a new unknown variable (pressure in the liquid) which is absent in Eqs. (1)-(3), can be neglected in the stability analysis.

The average force of interaction between the particles in unit volume of the mixture with liquid comprises the force of hydraulic interaction, the gravity force including the buyancy effect, and the buoyancy force caused by accelerated motion of the particles, where the quadratic law is assumed for the hydraulic force of the interphase interaction [9]:

$$nf = \frac{d_0}{a} \rho \, K(\rho) \, uu \, .$$

Components of the effect of associated mass and the Faxen effect are neglected because of their smallness in comparison with these forces. It should be noted that this is not suitable for suspensions of relatively small particles, where the effect of the intertial force due to the effect of the associated mass is quite substantial. In the present work this situation is not considered as it only concerns a system of large particles.

According to [8] the rheological closing of the system of equations (1)-(3) can be expressed in the form

$$\frac{P_1}{d_1} = \rho \, G(\rho) \, \theta \, , \tag{4}$$

$$\frac{\mu_1}{d_1} = 4\rho \, (y^{-1} + 0.8 + 0.76y) \, \mu_0 \, , \tag{5}$$

$$y = G(\rho) - 1 \, ,$$

$$\mu_0 = \frac{5}{48} a \, (\pi \, \theta)^{1/2} \, .$$

The function $G(\rho)$ describing steric interaction of the particles can be expressed in two versions, first by Carnahan-Starling's version [11]:

$$G(\rho) = \frac{1 + \rho + \rho^2 - \rho^3}{(1 - \rho)^3} \, , \tag{6}$$

and, second, by the version for a system of spherical particles near the state of dense packing following from Enskog's theory of dense gases [12]

$$G(\rho) = \frac{1}{1 - \left(\frac{\rho}{\rho_*} \right)^{1/2}} \, , \tag{7}$$

where ρ_* is the concentration of the randomized state of dense packing.

Certain strengths and weaknesses of these approaches to determination of $G(\rho)$ for various concentrations are shown in [13]. In what follows both versions are used.

The complete formulation of the problem of linear stability contains all the conservation equations (1)-(3), where the temperature θ is an unknown function. The approximate approach based on considering the equilibrium state of the medium can be useful in this case. This state is physically similar to thermodynamic equilibrium of molecular gas, where all parameters of the flow except pressure are homogeneous. This idealization allows us to make some substantial simplifications reported in [8]. Moreover, if it is possible to neglect the convective derivative in the left-hand side of Eq. (3) and the work done against the pressure forces of the pseudogas particles in comparison with the source terms, $\theta = \theta^*$, which suggests that the temperature of fluctuations in heterogeneous states is equal to the known temperature in locally homogeneous states. In this case the mass and momentum equations (conservation equations of dispersed phase) are sufficient for analysis of the stability, which decreases the order of the characteristic equation and, consequently, simplifies the study of singularities of the disturbance waves.

Conditions of neutral stability will be determined accordingly for all determinations of the effective temperature of fluctuations, first for the equilibrium state, with Eq. (3). In this case the temperature is considered as the temperature of a fictitious homogeneous state of the disperse medium characterized by local average values of dynamic variables. Second, for the situation in which the temperature is assumed to be equal to a known value corresponding to locally homogeneous states, the function of average characteristics of average motion of the disperse system and physical parameters is

$$\theta = \theta^* \tag{8}$$

and, finally, in the limiting case of a homogeneous flow with concentration ρ_0. The effective temperature of fluctuations is assumed to be constant for this state and is a function of the concentration ρ_0 only

$$\theta = \theta_0 . \tag{9}$$

In [9] for the temperature $\theta = \theta^*$ with the assumption of statistical independence of particles and isotropy of their fluctuations due to interparticle collisions, the following formula was obtained:

$$\theta^* = R(\rho, u)(u\varepsilon)^2 .$$

Here

$$R(\rho, u) = 1.17 \cdot 10^{-3} M(\rho, u)^2 \frac{\langle \rho'^2 \rangle}{\varepsilon^2},$$

$$M(\rho, u) = \frac{1}{\varepsilon} + \frac{1}{2}\left(\frac{d \ln K(\rho)}{d\rho} + \frac{\kappa a g \sigma}{K(\rho) u^2}\right),$$

$$\sigma = 1 - \frac{1}{\kappa}.$$

An undisturbed homogeneous flow is described by a single relation determined from Eq. (2):

$$u_0^2 = \frac{\kappa a \varepsilon_0 \sigma g}{K_0}, \quad K_0 = K(\rho_0),$$

where u_0 is the average velocity of the interphase slip.

In the coordinate system of the average motion of the dispersed phase, from Eq. (1) we have

$$v = \frac{\varepsilon_0 u_0 - \rho w}{\varepsilon}, \quad u = v - w = \frac{\varepsilon_0 u_0 - w}{\varepsilon},$$

$$u' = \frac{u_0 \rho' - w'}{\varepsilon_0}.$$

Then the following expressions can be written for θ^* and θ_0

$$\theta^* = R(\rho, u)(\varepsilon_0 u_0 - w)^2,$$

$$\theta_0 = R_0 (\varepsilon_0 u_0)^2, \quad R_0 = 1.17 \cdot 10^{-3} M_0^2 \frac{\langle \rho'^2 \rangle |_0}{\varepsilon_0^2},$$

$$M_0 = \frac{1}{2}\left(\frac{3}{\varepsilon_0} + \left.\frac{d \ln K}{d\rho}\right|_0\right).$$

After linearization of Eqs. (1)-(3) in perturbations of the concentration and velocity of particles we obtain a system of equations for disturbances of the homogeneous flow

$$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial w'}{\partial x} = 0 , \tag{10}$$

$$\rho_0 \frac{\partial w'}{\partial t} = - \theta_0 \frac{\partial (\rho G(\rho))}{\partial \rho} \bigg|_0 \frac{\partial \rho'}{\partial x} - \rho_0 G_0 \frac{\partial \theta'}{\partial x} +$$

$$+ \frac{\mu_1}{d_1} \frac{\partial^2 w'}{\partial x^2} + \rho_0 \varepsilon_0 \sigma g \left(\frac{d \ln K}{d\rho} \bigg|_0 + \frac{3}{\varepsilon_0} \right) \rho' - 2\rho_0 \sqrt{\left(\frac{K_0 \sigma g}{\kappa a \varepsilon_0} \right)} w' + \rho_0^2 \frac{\partial w'}{\partial t} + \rho_0 \varepsilon_0 \frac{\partial w'}{\partial t} , \tag{11}$$

$$G_0 = G(\rho_0) ,$$

$$\frac{3}{2} \frac{\partial \theta'}{\partial t} = - G_0 \theta_0 \frac{\partial w'}{\partial x} + 2 \left[\left\{ \left(\frac{a \varepsilon_0 \sigma g}{\kappa K_0} \right)^{1/2} \varepsilon_0 \frac{dR}{d\rho} \rho' - 2R_0 (1 - \varepsilon_0^2) w' \right\} \varepsilon_0^2 \sigma g - \left(\frac{K_0 \varepsilon_0 \sigma g}{\kappa a} \right)^{1/2} \theta' \right] , \tag{12}$$

where for the disturbance of the last term in the right-hand side of Eq. (3) we have

$$\left(\sqrt{\theta} (\sqrt{\theta^*} - \sqrt{\theta}) \right)' = \left(\sqrt{\theta_0} (\sqrt{\theta^*} - \sqrt{\theta}) \right)' \approx \sqrt{\theta_0} (\sqrt{\theta_0 + \theta^{*'}} - \sqrt{\theta_0 + \theta'}) =$$

$$= \frac{\sqrt{\theta_0}}{2 \sqrt{\theta_0}} (\theta^{*'} - \theta') = \frac{1}{2} (\theta^{*'} - \theta') .$$

It is convenient here to use dimensionless variables

$$t_1 = t \Big/ \left(\frac{a\kappa}{g} \right)^{1/2} , \quad x_1 = x/(a\kappa) , \quad w_1 = w'/(g\kappa a)^{1/2} , \quad \rho_1 = \rho' , \quad \theta_{01} = \theta_0/(ga\kappa) .$$

Writing the system of equations (10)-(12) in these variables and considering the simplest disturbance wave, we obtain the characteristic equation for the dimensionless frequency $\omega_1 = \omega/(g/ak)^{1/2}$ in the form

$$\omega_1^3 + i d_{11} \omega_1^2 + (c_{22} + i d_{22}) \omega_1 + c_{33} + i d_{33} = 0 , \tag{13}$$

$$d_{11} = 2d_1^0 + \frac{4}{3} (K_0 \varepsilon_0 \sigma)^{1/2} ,$$

$$c_{22} = - c_2^0 - \frac{2}{3} \left(4d_1^0 (K_0 \varepsilon_0 \sigma)^{1/2} + \frac{\theta_{01}}{F} G_0^2 k_1^2 \right) ,$$

$$d_{22} = \frac{8}{3} R_0 (1 - \varepsilon_0^2) \frac{G_0 \varepsilon_0^2 \sigma}{F} k_1 - d_2^0 ,$$

$$c_{33} = \frac{4}{3} (K_0 \varepsilon_0 \sigma)^{1/2} d_2^0 ,$$

$$d_{33} = - \frac{4}{3} (K_0 \varepsilon_0 \sigma)^{1/2} \left(c_2^0 + \frac{\sigma \varepsilon_0^2 \rho_0}{K_0 F} G_0 \frac{dR}{d\rho} \bigg|_0 k_1^2 \right) ,$$

$$d_1^0 = \frac{\sqrt{\sigma}}{2F} \left(2 \sqrt{\left(\frac{K_0}{\varepsilon_0} \right)} + \frac{5}{12} \sqrt{\left(\frac{\pi R_0 \varepsilon_0^2}{K_0} \right)} (y^{-1} + 0.8 + 0.76y) \bigg|_0 k_1^2 \right) ,$$

$$c_2^0 = \frac{R_0 \varepsilon_0^3 \sigma}{FK_0} \left. \frac{d\,(\rho G\,(\rho))}{d\rho} \right|_0 k_1^2 ,$$

$$d_2^0 = \frac{2\rho_0 \varepsilon_0 \sigma}{F} M_0 k_1 ,$$

$$F = \varepsilon_0 \sigma .$$

It is easy to understand that the number of imaginary roots of Eq. (13) for a certain fixed κ corresponds to the number of transitions of stability intervals (or vice versa) into intervals unstable in ρ_0. In order to determine the boundaries of these intervals by the standard procedure, the signs of the minors of the second, fourth, and sixth orders of the matrix composed of the coefficients of Eq. (13) are investigated. Simple but tedious calculations have revealed that the minors of the second and sixth order do not change signs within the whole concentration range, while the minor of the fourth order changes sign and gives the following condition of instability:

$$d_{22}^2 / d_{11} + d_{11} c_{22} - d_{33} > 0 . \tag{14}$$

For approximation (8) $\theta = \theta^*$, the characteristic equation is quadratic,

$$\omega_1^2 + 2\,(c_1^* + i d_1^*)\,\omega_1 - (c_2^* + i d_2^*) = 0 , \tag{15}$$

where

$$c_1^* = \frac{\sqrt{\sigma}}{F} G_0 \sqrt{\left(\frac{\varepsilon_0^3}{K_0} \right)} R_0\,(1 - \varepsilon_0^2)\,k_1 ;$$

$$d_1^* = d_1^0 ; \quad d_2^* = d_2^0 ;$$

$$c_2^* = c_2^0 + \frac{\rho_0 \varepsilon_0^3 \sigma}{FK_0} G_0 \left. \frac{dR}{d\rho} \right|_0 k_1 .$$

At real wave numbers k_1 the real and imaginary components of the complex frequency ω_1 are written as

$$\begin{gathered} \mathrm{Re}\,\omega_1 = -\,c_1^* \pm \left\{ \frac{1}{2} A + \frac{1}{2} (A^2 + B^2)^{1/2} \right\}^{1/2} , \\ \mathrm{Im}\,\omega_1 = -\,d_1^* \pm \left\{ -\frac{1}{2} A + \frac{1}{2} (A^2 + B^2)^{1/2} \right\}^{1/2} , \\ A = c_1^{*2} - d_1^{*2} + c_2^* , \quad B = 2 c_1^* d_1^* + d_2^* . \end{gathered} \tag{16}$$

It can be seen from (16) that root (15) corresponding to the upper sign in the definition of Im ω_1 in Eq. (16) is the most critical as regards violation of stability and for this case the instability condition is of the following form:

$$(2 c_1^* d_1^* + d_2^*)^2 > 4 d_1^{*2}\,(c_1^{*2} + c_2^*) . \tag{17}$$

A similar expression for the instability criterion can be obtained for the simplest case of Eq. (9), where $\theta = \theta_0$. Omitting intermediate calculations we obtain

$$d_2^{02} > 4 d_1^{02} c_2^0 . \tag{18}$$

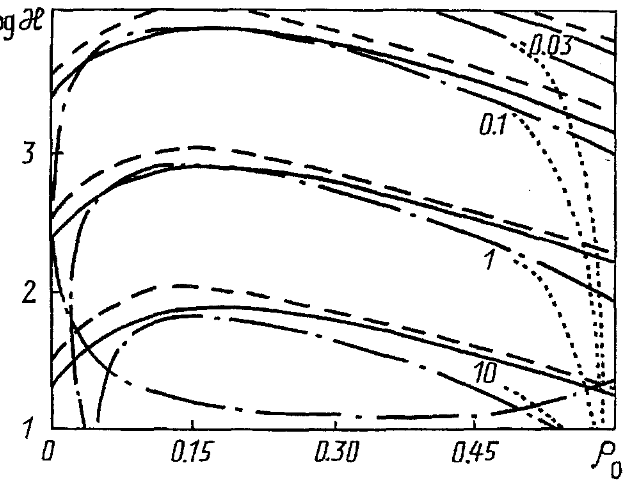


Fig. 1. Curves of neutral stability at various scale factors N (figures near the curves) according to (14), (15), and (18); the solid line corresponds to condition (17); the dashed line, to (18); the dotted-dashed to (14); the dotted, to definition (17) following Enskog's theory (7) in (4) and (5).

We will consider stability of the limited flow for which the wave number cannot be less than a certain limiting value $k^* \simeq l^{-1}$, where l is the scale of the flow. We will introduce the scale factor N following naturally from the relations

$$\frac{k_{1*}}{\kappa} = N\kappa , \quad N = k_*^2 a^2 , \quad k_{1*} = a\kappa k_* \sim \frac{a}{l}\kappa . \tag{19}$$

Substitution of the complex $N\kappa$ for the factor k_1^2/κ in Eqs. (14), (17), and (18) gives similar inequalities.

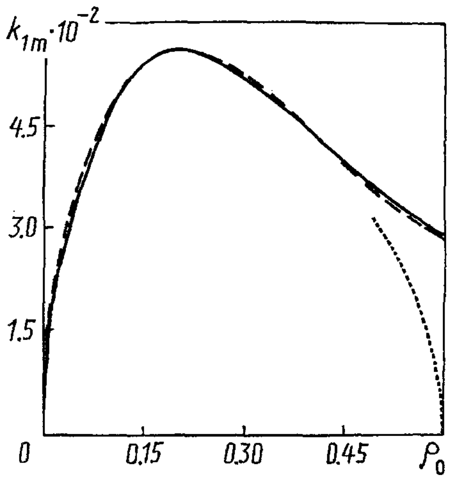
Figure 1 shows families of curves of neutral stability obtained from Eqs. (14), (17), and (18) with Eq. (19) for different values of the scale factor N. For $\theta = \theta^*$ and $\theta = \theta_0$ the regions of instability lie under these curves, while the model comprising all the three conservation equations has the region of instability limited by the lower branch of the curve of neutral stability which, unlike the first two curves, is loop-shaped. The dots also indicate the branches of the curves of neutral stability at high concentrations obtained by using the expression from [7] in calculations. The narrow region of stability lies to the right of these curves up to $\rho_* = 0.6$.

It can be seen that as the flow dimension decreases (the parameter N increases), the stability region expands. It follows from the analysis of the curves in Fig. 1 that instability increases in the case of installations of large overall size with large linear scales, which is important in simulation of industrial installations.

It should be noted that the present calculations also confirm the conclusion of some experimental studies [1, 3, 4] about flow stability in the range of high concentrations up to dense packing. In calculations this effect is more pronounced when Eq. (7) is taken into consideration in Eqs. (4) and (5). It can be seen that determining the function $G(\rho)$ following the model of a smoothed volume in Enskog's theory results in the stability region adjacent to the densly packed state of the system for all the values, including those corresponding to overall sizes of installations as large as one desires.

For particles suspended in gas, κ of the order of magnitude of 1000 and more, the curves obtained from Eqs. (8) and (9) (for the case $\theta = \theta^*$ and $\theta = \theta_0$, respectively) and the upper branch of the curves calculated for the general case (with Eq. (3) for θ) almost coincide. Therefore, in the initial analysis of stability for gas suspensions only the relatively simple models $\theta = \theta^*$ and $\theta = \theta_0$ can be used without substantial loss of accuracy in the calculations, which is especially important for engineering calculations. Because of this, it is possible to obtain sufficiently detailed information on the waves of maximum growth of such systems as well.

In Fig. 2 the dimensionless wave number k_{1m} corresponding to maximum increment of growth of disturbances is plotted versus ρ_0. This increment corresponds to the maximum Imω_1 in Eq. (16). In Fig. 3 the dimensionless frequency of disturbances Re ω_1 in the instability region is plotted versus ρ_0 for corresponding



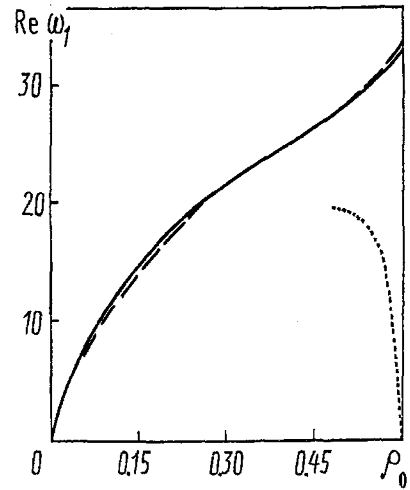


Fig. 2. Plot of the dimensionless wave number versus ρ_0 maximizing Im ω_1 from (16); the solid line corresponds to the definition of the effective temperature according to (8); the dashed line, to Eq. (9); the dotted line corresponds to definition (8) following Enskog's theory (7) in (4), (5).

Fig. 3. Plot of Re ω_1 versus ρ_0 in the region of instability, whose growth increment is maximum; the solid line corresponds to the definition of the effective temperature according to (8); the dashed line correspond to the definition by (9); the dotted line corresponds to definition (8) following Enskog's theory (7) in (4), (5).

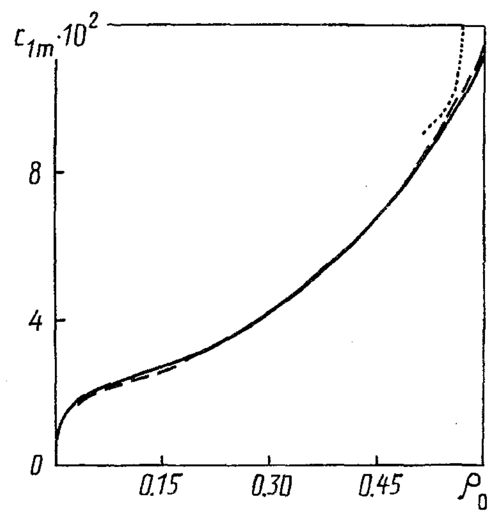


Fig. 4. Plot of dimensionless velocity of propagation of maximum growth waves c_{1m} versus ρ_0; the solid line corresponds to definition (8) of the effective temperature according to (8); the dashed line, to definition (9); the dotted line corresponds to definition (8) following Enskog's theory (7) in (4), (5).

k_{1m}. In Fig. 4 the dimensionless velocity c_{1m} = [Re $\omega_1(k_{1m})$]$/k_{1m}$ of propagation of maximum growth waves is plotted versus ρ_0. In these figures the curves almost coinciding with model (8) are also shown for the case of Eq. (9) $\theta = \theta_0$.

All the curves are plotted for the value of κ characteristic of gas suspensions, which was taken to be 1000 in the calculations.

The curve $k_{1m}(\rho_0)$ has a maximum in the range of concentrations close to ρ_0 = 0.2. The most probable dimension of the bubbles initially formed in the unrestricted flow corresponds to it. As ρ_0 increases, the size of gas cavities arising in the layer as a quantity inversely proportional to the wave number k_{1m} should grow. The same can also take place in the range of small concentrations at $\rho_0 < 0.2$. Up to concentrations $\rho_0 = 0.45–0.5$, the wave frequency ω_1 is an increasing function of ρ_0. In the range of high concentrations, as the state approaches dense packing, where determination of $G(\rho)$ by formula (7) based on Enskog's theory is more reasonable, the wave frequency decreases rapidly as the concentration rises. However, the rate of this decrease is lower than the rate of decrease in the wave number, which results in the appropriate increase in the velocity of propagation of the waves of maximum growth tending to infinity in the state of dense packing.

In conclusion, it should be noted that stabilization of disturbances at small κ characteristic of the particle suspensions in liquids is caused by the effect of viscous dissipation of the fluctuation energy in the flow induced by the hydrodynamic resistance to the fluctuation motion of particles which is included in the fluctuation energy transfer equation (3). However, in an infinite flow the stabilizing effect of viscous dissipation for these systems is important and the flow inevitably appears stable except for the cases where κ is close to 1. However, it cannot be concluded that the results of calculation of disturbance characteristics by models (8) and (9), on the hand, and by the general model including all three conservation equations (1)-(3), on the other, are adequate, as was expected. This requires an additional series of calculations with characteristic equation (13) obtained for the equilibrium state with Eq. (3), which can be the object of an independent study.

NOTATION

a, particle radius; f, hydraulic force exerted on particles by liquid; $k(\rho)$, function introduced in (2); $G(\rho)$, function introduced in (4); k, wave number; m, particle mass; n, numerical concentration of particle; P_1, particle pressure; v, liquid velocity; w, particle velocity; u, relative velocity of the liquid; x, vertical coordinate; ε, porosity of the bed of particles; ρ, volume concentration of the dispersed phase; ω, frequency; θ, effective temperature of the gas of particles; μ_1, viscosity of the gas of particles; ξ, resistance coefficient of a particle; $\langle\rho'^2\rangle$, variance of fluctuation of the concentration; Re ω, Im ω, real and imaginary components of the frequency; N, scale factor introduced in (19); d_{11}, d_{22}, c_{22}, c_{33}, d_{33}, d_1, d_2, c_2, coefficients introduced in (13) and (16). Subscripts: *, refers to parameters corresponding to equilibrium states; 0, refers to parameters of homogeneous states; ', refers to fluctuations; 1, refers to dimensionless quantities.

REFERENCES

1. R. L. Pigford and T. Baron, Ind. Eng. Chem. Fund, **1**, 81-87 (1965).
2. O. Molerus, Chem. Ing. Technic., **39**, 341-348 (1967).
3. S. K. Gard and J. W. Pritchett, J. Appl. Phys., **46**, 4493-4500 (1975).
4. G. M. Homsy, M. M. El-Kaissy, and A. K. Didwania, Int. J. Multiphase Flow, **6**, 305-318 (1980).
5. G. K. Batchelor, J. Fluid Mech., **193**, 75-100 (1988).
6. A. K. Didwania and G. M. Homsy, Ind. Eng. Chem. Fund, **20**, 318-323 (1981).
7. D. J. Needham and J. N. Merkin, J. Fluid Mech., **131**, 427-454 (1983).
8. Yu. A. Buevich and A. G. Rubtsov, Inzh.-Fiz. Zh., **63**, No. 4, 414-424 (1992).
9. Yu. A. Buevich and Sh. K. Kapbasov, Inzh.-Fiz. Zh., **65**, No. 1, 48-56 (1993).
10. Yu. A. Buevich and Sh. K. Kapbasov, Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza, No. 6, 57-66 (1993).
11. N. F. Carnahan and K. E. Starling, J. Chem. Phys., **51**, 635-637 (1969).
12. J. O. Hirschfelder, C. F. Curtis, and B. Bird, Molecular Theory of Gases and Liquids, Wiley, N. Y.; Chapman & Hall, London, (1954).
13. Yu. A. Buevich, Inzh.-Fiz. Zh., **65**, No. 1, 39-47 (193).